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EVALUATION OF ENERGY RELEASE RATE IN THE CRACK–MICROCRACK INTERACTION PROBLEM

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Abstract—The crack–microcrack array interaction problem, in terms of the statistical distribution of microcrack length, density and orientation, is formulated in this paper. The formulation is based on Green's function of a dislocation dipole placed in the vicinity of the main crack tip. The energy release rate (ERR) associated with the microcrack array evolution is also formulated in terms of the distribution and their gradients. The effect of the microcrack density, length and orientation on stress intensity factors and ERRs are illustrated on examples.

1. INTRODUCTION

A damage zone (DZ) usually accompanies slow crack propagation under fatigue and creep conditions. In this paper we consider a special case of a damage zone consisting of an array of localized discontinuities such as microcracks or crazes. Figure 1 illustrates an array of crazes formed in the vicinity of a fatigue crack in an amorphous polymer. Statistical distributions of microcrack densities, orientation and length appear to be the most appropriate characterizations of such damage (Chudnovsky and Wu, 1991; Huang *et al.*, in press). A hypothesis of a similarity of DZ, i.e. self-similarity of the statistical distributions, in the process of the DZ evolution has been first proposed theoretically (Chudnovsky, 1984) and then supported by experimental examinations (Botsis and Kunin, 1987; Zhang, 1990). The self-similarity hypothesis (SSH) yields a decomposition of the DZ propagation into elementary movements such as translation, rotation and deformation. The corresponding driving forces, in accordance with the general framework of the thermodynamics of irreversible processes, are represented by linear functions of the energy release rates (ERR) associated with the elementary movements (Chudnovsky, 1984). This motivates the present study of crack–microcrack array interaction and an evaluation of ERRs resulting from the array translation, expansion etc.

Three approaches have been recently advanced to evaluate elastic fields associated with the presence of microcrack array in the vicinity of the main crack tip. In the first approach the microcrack array is modeled by an inclusion of an effective elastic medium. This well-posed boundary value problem of a crack partially penetrating into a "softer inclusion" has been addressed by various authors (Steiff, 1987; Ortiz, 1987; Huchinson, 1987; Wu, 1988). However, there are various shortcomings in this approach from the physics standpoint. First of all it does not account for local fluctuations of microcrack density and length, which is of primary importance for the fracture process. Secondly, the relation between the statistics of the microcrack array in the vicinity of the main crack and an effective elastic constant is, in general, unknown. Determination of such a relation is equivalent to solving the crack–microcrack interaction problem. In addition to that the distribution of microcracks in the array is usually a heterogeneous one. To reflect that, an equivalent elastic inclusion should be nonhomogeneous and anisotropic. It leads to certain computational difficulties.

Another approach to crack–microcrack interaction is based on a detailed description of the location, size and orientation of every microcrack in every particular realization of the microcrack array (Chudnovsky and Kachanov, 1983; Horii and Nemat-Nasser, 1983; Kachanov, 1985; Rose, 1986; Rubinstein, 1986; Chudnovsky *et al.*, 1987). Apparently, this leads to a computational limitation and the method becomes impractical for an array similar to the one shown in Fig. 1.

In the third approach the microcrack array is characterized by statistical distributions of microcrack densities, sizes and orientation. It leads to evaluation of integral (average) parameters associated with the microcrack array (Chudnovsky and Qoezdou, 1988; Chudnovsky and Wu, 1990; Chudnovsky and Wu, 1991; Wu and Chudnovsky, 1990). The present paper follows the third approach and is a continuation of our previous work.

The statistical distribution of the microcrack length as well as the distance between the microcracks, their locations and orientations with respect to the main crack are essential for the interaction problem. We employ a new characterization of a random array of microcracks in terms of distributions of the size, orientation and density of microcracks proposed by Chudnovsky and Wu (1991) and characterize the damage $D(\mathbf{x})$ at a given point x by the microcrack density $\rho_0(\mathbf{x})$, angular distribution $\varphi(\theta/\mathbf{x})$ and the microcrack length distribution $p(l/\mathbf{x}, \theta)$. The microcrack density $\rho_0(\mathbf{x})$ is defined as 1/2 of crack surfaces per unit volume and has dimension m^2/m^3. Apparently, it is different from the dimensionless "microcrack density" ε conventionally used in damage mechanics. A corresponding quantity in our case is microcrack concentration ρl. The relationship between ρl and the effective elastic constants for "dilute" microcrack concentration in the 2-D case can be easily found [Wu and Chudnovsky, (1990)].

Two issues are addressed in this paper. The first is a formulation of the crack–microcrack array interaction problem in terms of the above distributions and its solution. It is based on Green's function of a dislocation dipole placed in the vicinity of the main crack tip given by Ballarini and Denda (1988). The second is an evaluation of ERRs. The effect of the distributions of microcrack density, length and orientation on the stress intensity factor (SIF) and the ERRs due to a microcrack array is illustrated in examples.

2. ELASTIC INTERACTION OF A CRACK WITH A RANDOM ARRAY OF MICROCRACKS

2.1. *Formulation of the problem*

The linear elastic interaction of a crack microcrack array can be obtained by the superposition method based on Green's function G for a dislocation dipole interacting with a crack. Microcrack opening displacement is conventionally represented by a continuous distribution $\mathbf{b}(\xi)$ of dislocation dipoles. Thus the stress, displacement and SIF of the main crack due to a particular microcrack (l_0) can be expressed as:

$$\mathbf{u}(\mathbf{x}) = \int_{l_0} \mathbf{b}(\xi)\mathbf{\Phi}(\mathbf{x},\xi)\,d\xi,\quad \boldsymbol{\sigma}(\mathbf{x}) = \int_{l_0} \mathbf{b}(\xi)\mathbf{F}(\mathbf{x},\xi)\,d\xi,\quad K = \int_{l_0} \mathbf{b}(\xi)\mathbf{G}_{SIF}(\xi)\,d\xi, \tag{1}$$

where the influence functions $\mathbf{\Phi}$, $\mathbf{F}$ and $\mathbf{G}_{SIF}$ are known functions, obtained by simple transformations of Green's function G (Wu and Chudnovsky, 1991). Then by means of the superposition principle, the stress, displacement and SIF due to a microcrack array can be obtained by integrating (1) over the domain V occupied by the microcrack array with microcrack density ρ as a weight function:

$$\mathbf{u}^{A}(\mathbf{x}) = \int_V \rho(\xi)\mathbf{b}(\xi)\mathbf{\Phi}(\mathbf{x},\xi)\,d\xi,\quad \boldsymbol{\sigma}^{A}(\mathbf{x}) = \int_V \rho(\xi)\mathbf{b}(\xi)\mathbf{F}(\mathbf{x},\xi)\,d\xi$$

$$K^{A} = \int_V \rho(\xi)\mathbf{b}(\xi)\mathbf{G}_{SIF}(\xi)\,d\xi. \tag{2}$$

The integrals in (2) are well defined if the microcrack concentration ρl tends to zero faster than $r^{1/2}$:

$$\lim_{r\to 0} r^{-1/2}\rho(r)l(r) = 0. \tag{3}$$

We consider below the case when this condition is satisfied.

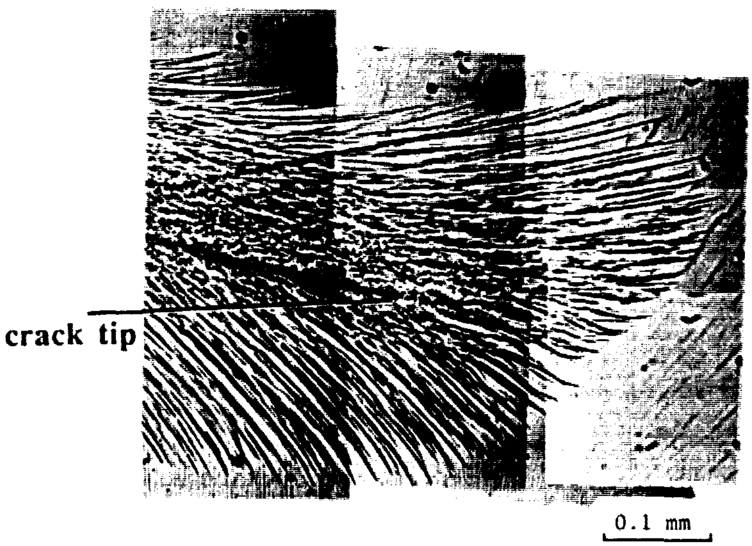

Fig. 1. The optical micrograph displaying the damage zone (craze array) near the crack tip in an amorphous polymer.

The elastic fields in a vicinity of the main crack surrounded by a process zone can be expressed as a sum:

$$\mathbf{u} = \mathbf{u}^{c}+\mathbf{u}^{A}, \quad \boldsymbol{\sigma} = \boldsymbol{\sigma}^{c}+\boldsymbol{\sigma}^{A}, \quad K = K^{0}+K^{A} \tag{4}$$

where $\mathbf{u}^c$, $\boldsymbol{\sigma}^c$ and K^0 are the diplacement, stress and SIF due to the main crack only under the remote loading σ^0, respectively. Thus the traction-free condition on the main crack is met since both terms in (4) satisfy it, the remote loading boundary condition is satisfied by the first terms in (4). The remaining boundary conditions, i.e. the traction-free faces of the microcracks, are met by solving a system of corresponding singular integral equations.

The equations are written for every microcrack embedded into an effective stress field. The latter is defined as follows. Let us consider the effective stress $\boldsymbol{\sigma}^{\text{eff}}$ along the ith microcrack line generated by the main crack and the rest of the microcracks in the absence of the ith microcrack:

$$\boldsymbol{\sigma}^{\text{eff}}(\mathbf{x}) = \boldsymbol{\sigma}^{c}(\mathbf{x})+\sum_{j\neq i}^{M}\int_{l^{(j)}} \mathbf{b}^{(j)}(\xi)\mathbf{F}(\mathbf{x},\xi)\,\mathrm{d}\xi. \tag{5}$$

Then applying $-\boldsymbol{\sigma}^{\text{eff}}(\mathbf{x})\cdot\mathbf{n}(\mathbf{x})$ on ith microcrack faces, we satisfy the traction free requirement for the ith microcrack. Applying this treatment to every microcrack, one obtains the system of integral equations. For simplicity, we assume $\boldsymbol{\sigma}^{\text{eff}}(\mathbf{x})$ being constant on the microcrack scale. Then the relation between the effective stress and the microcrack opening displacement is well known:

$$\mathbf{b} = \frac{\pi l}{E}\boldsymbol{\sigma}^{\text{eff}}\cdot\mathbf{n}. \tag{6}$$

Then, combining (5) and (6), we obtain a system of integral equation with respect to the unknown $\mathbf{b}$ functions. Solution of these equations leads to the solution of the interaction problem.

Substituting the summation in (5) by the integration over V with weight function $\rho(\mathbf{x})$ and employing a conventional regularization of the singular integrals in (2) (Chudnovsky *et al.*, 1987), we rewrite eqns (5) as:

$$\boldsymbol{\sigma}^{\text{eff}}(\mathbf{x}) = \boldsymbol{\sigma}^{c}(\mathbf{x})+\int_{V}\rho(\xi)[\mathbf{b}(\xi)-\mathbf{b}(\mathbf{x})]\mathbf{F}(\mathbf{x},\xi)\,\mathrm{d}\xi. \tag{7}$$

Finally, combining eqns (6) and (7), we obtain the following equation to determine the unknown microcrack opening displacement vector $\mathbf{b}(\mathbf{x})$:

$$\mathbf{b}(\mathbf{x}) = \frac{\pi l(\mathbf{x})}{E}\boldsymbol{\sigma}^{c}\cdot\mathbf{n}+\frac{\pi l(\mathbf{x})}{E}\left\{\int_{V}\rho(\xi)[\mathbf{b}(\xi)-\mathbf{b}(\mathbf{x})]\mathbf{F}(\mathbf{x},\xi)\,\mathrm{d}\xi\right\}\cdot\mathbf{n}. \tag{8}$$

Apparently from (8) the components b_k of an average vector opening $\mathbf{b}(\mathbf{x})$ at point $\mathbf{x}$ can be viewed as the sum of the opening due to the main crack with remote load and due to the microcrack array in the presence of the main crack. The integral in (8) can be discreted into summation by employing the same method as Chudnovsky and Wu (1991). Thus, (8) can be reduced into two algebraic equations in terms of the components of microcrack opening displacement b_1 and b_2:

$$\begin{aligned} \lambda_{11}B_1+\lambda_{12}B_2 &= F_1\\ \lambda_{21}B_1+\lambda_{22}B_2 &= F_2, \end{aligned} \tag{9}$$

where B_1 and B_2 are the column matrices consisting of the values of microcrack opening b

at points of discretization. Matrix λ_{ij} and F_i are known functions of the microcrack density, length distribution and the elastic properties of the undamaged material. Equation (9), in principle, can be solved by a numerical technique. However, the singularity of effective stress near the crack tip creates an obstacle for the computation. To overcome this problem, we decompose the effective stress $\boldsymbol{\sigma}^{\text{eff}}$ into singular and regular components:

$$\boldsymbol{\sigma}^{\text{eff}}(\mathbf{x}) = \frac{K^{\text{eff}}}{\sqrt{2\pi r}}\varphi(\theta) + \boldsymbol{\sigma}_0^{\text{eff}}(\mathbf{x}). \tag{10}$$

The form of the singular part of the effective stress is based on the analytical solution with unknown K^{eff}. The regular part of the effective stress is obtained numerically. If the microcrack orientation is statistically isotropic at every point of the active zone, the effective stress singularity is expected to be the same as the conventional singularity in an isotropic material. For an anisotropic statistics i.e. for a microcrack array with a dominant orientation, the singularity of the stress field is expected to resemble that in a medium with a corresponding anisotropy. Then, the order of singularity is still the same ($r^{-1/2}$), but the angular distribution of stress $\varphi(\theta)$ will depend on the particular anisotropy.

Knowing the $\mathbf{b}(\mathbf{x})$ field, the SIF $K_{\text{I}}^{\text{eff}}$, the elastic fields $\mathbf{u}(\mathbf{x})$ and $\boldsymbol{\sigma}(\mathbf{x})$ can be radially reconstructed by means of (2) and (4).

2.2. *Crack–microcrack array interaction*

Example 1. In this example we compare the SIF from our scheme with that obtained by modeling a microcrack array by an elastic inclusion with isotropic effective elastic properties. A circular shape damage zone and the corresponding inclusion are considered. The elastic properties of the inclusion are chosen as effective properties of an elastic medium perforated by a microcrack array with constant microcrack concentration (ρl = constant). For computational purposes we select the ratio of the radius of damage zone and the main crack length $R/L = 0.1$. All of the microcracks are parallel to the main crack, so the singular part of the resulting effective stress field is similar to that for an orthotropic material where the orthotropic property comes from the distribution of microcracks (Wu and Chudnovsky, 1990). A low microcrack density case, the range $0 < \rho l < 0.2$, is considered, to examine the effect of a microcrack array on the SIF. It should be noted that the above microcrack concentration is different from commonly used microcrack densities ε (Huchinson, 1987); here the relation between the effective elastic properties and the microcrack concentration is taken from Wu and Chudnovsky (1990) for the two-dimensional case. Considering eqn (2), the effective SIF can be expressed as follows:

$$K^{\text{tot}} = K^0 + K^A = K^0 + \int_V \rho\mathbf{b}(\xi)\mathbf{G}_{\text{SIF}}(\xi)\,\mathrm{d}\xi. \tag{11}$$

The dependence of SIF on microcrack concentration ρl is shown in Fig. 2 by solid line. The dotted line represents the SIF of Huchinson (1987) whose result given in terms of the ratio of the initial and effective Young's modulus of material has been reformulated in terms of the microcrack concentration. It should be emphasized that the Huchinson (1987) result is obtained for an isotropic inclusion which can be considered as a model of an isotropically distributed microcrack array. For the case when all microcracks are parallel to the main crack, our solution should be compared with an anisotropic inclusion problem. However, to our knowledge, the SIF for a crack partially penetrating an anisotropic inclusion is not known, therefore we compare our results with the closest available solution. It is expected that shielding of a parallel microcrack array is higher than that of randomly distributed microcracks, i.e. $K_{\text{I}}^{\text{eff}}$ of our solution is smaller than $K_{\text{I}}^{\text{eff}}$ given by Huchinson (1987).

The main advantage of the method described above is that it can deal with crack-damage interaction equally well for uniform and nonuniform distributions of microcracks. As soon as the microcrack density $\rho(\mathbf{x})$ and the microcrack length distribution $l(\mathbf{x})$ are

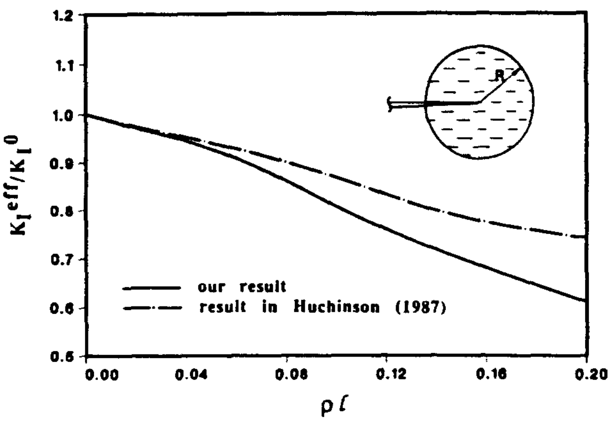

Fig. 2. The dependence of K_I^{eff} on the microcrack concentration ρl.

given (e.g. measured by experimental means), the interaction problem can be solved using the same numerical procedure as above. This statement is illustrated in the next example.

Example 2. Let us consider a specimen of the same geometry and loading condition as in an experiment reported by Botsis (1988) (an SEN specimen of an amorphous polymer with Young's modulus $E = 2.2$ Gpa, Poisson's ratio $v = 0.3$, applied load $\sigma_{22} = 16$ Mpa). The evolution of the damage zone was monitored by a videorecording system attached to an optical microscope. It should be noticed that the damage reported by Botsis (1988) consists of crazes. In our example the crazes are substituted by microcracks. The calculation is performed for the microcrack array whose density coincides with the observed craze density, and the length distribution resembles that of crazes.

The microcrack density $\rho_0(\mathrm{x})$ employed is shown in Fig. 3a. The distribution of mathematical expectation of microcrack length is chosen as an extrapolation of the peripheral craze length distribution:

$$l(\mathrm{x}) = 0.06\left(\frac{x_1}{L_a}\right)^{\alpha} + 0.15\left(\frac{x_2}{w}\right)^{\alpha} \quad \text{if } x_1 > 0$$

$$l(\mathrm{x}) = 0.15\left(\frac{x_2}{w}\right)^{\alpha} \quad \text{if } x_1 < 0, \tag{12}$$

where l_a and w are the length and half width of the active zone respectively. Using the numerical procedure as described above the effective stress field is constructed for such a microcrack array. The results of numerical computation for $\alpha = 2$ are exemplified in Fig. 3b, which displays the σ_{22}^{eff} component of the effective stress field. The other component of effective stress as well as the microcrack opening distribution are reported in Chudnovsky and Wu (1991). The effective SIF K_I^{eff} in this case is $K_I^{eff} = 0.88\ K_I^0$, where K_I^0 stands for the SIF of the main crack with the absence of the damage zone.

3. EVALUATION OF ENERGY RELEASE RATES

An elegant approach of evaluating the elastic energy changes due to initiation and growth of defects was outlined by Eshelby (1956, 1975). Following his approach one can express the energy release associated with the process zone translation, expansion etc. in terms of Eshelby tensor P. For example, the ERR J_k due to "translation" of the damage zone can be written as:

$$J_k = \int_V \partial_j P_{kj}\,\mathrm{d}V; \quad P_{kj} = f\delta_{kj} - \sigma_{ji}u_{i,k}. \tag{13}$$

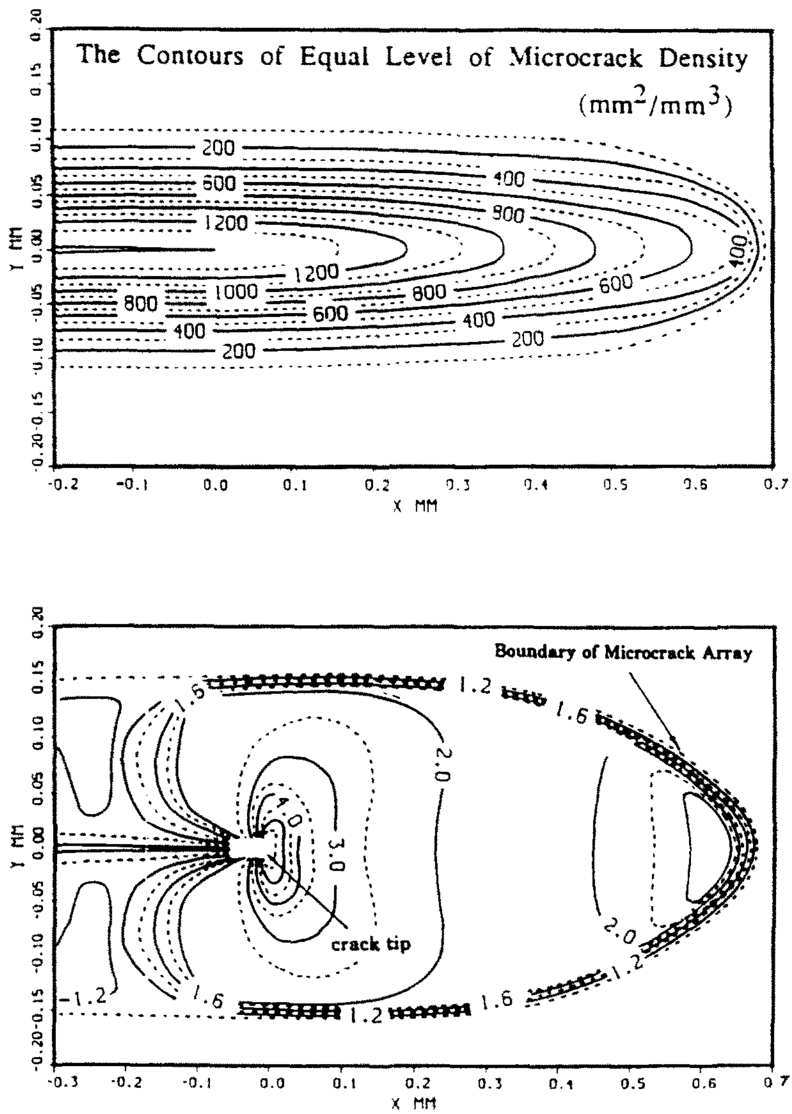

Fig. 3. (a) The contours of equal level of microcrack density $\rho_0(x)$ (mm^2/mm^3) in the active zone. The dot lines correspond to the intermediate of $\rho_0(x)$. (b) The contour of the equal value of effective $\sigma_{22}^{\mathrm{eff}}$ distribution normalized by σ_{22}^{0} in the damage zone for $\alpha = 2$.

Here V is the domain occupied by the damage zone and f is the strain energy density. To evaluate (13) one needs to know the elastic fields $\boldsymbol{\sigma}$ and $\mathbf{u}$ of the interaction problem discussed in the previous section.

Below we consider only the energy release associated with translation of the damage zone. Let us decompose the DZ into $N_\alpha \times N_\beta$ elementary cells (see Fig. 4). Then the integral over V in (13) can be rewritten as:

$$J_1 = \sum_{\alpha=1}^{N_\alpha} \sum_{\beta=1}^{N_\beta} \int_{V_{\alpha\beta}} \partial_j P_{1j}\, \mathrm{d}V, \tag{14}$$

where $V_{\alpha\beta}$ is the volume of the elementary cell. Since $\partial_j P_{1j} = 0$ within a homogeneous domain, the area integral in (14) can be converted into a path integral by means of Gauss Theorem:

$$\int_{V_{\alpha\beta}} \partial_j P_{1j}\, \mathrm{d}V = \int_{\Gamma_{\alpha\beta}} P_{1j} n_j\, \mathrm{d}\Gamma, \tag{15}$$

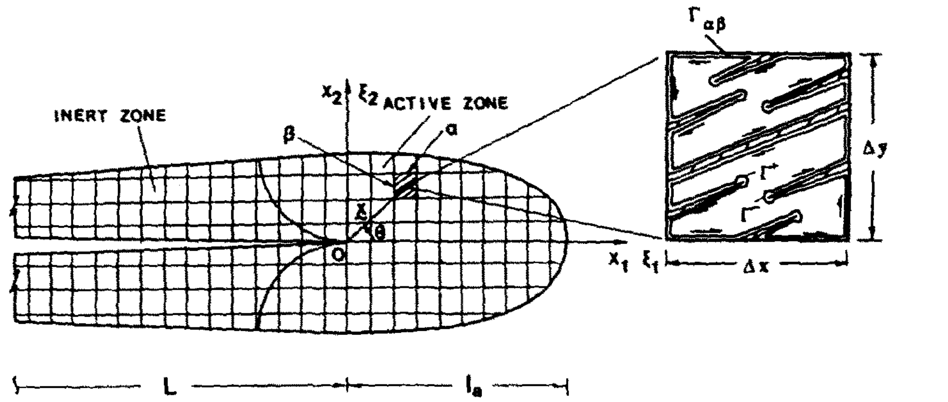

Fig. 4. The schematic representation of the damage zone and the subdivision of it into a set of elementary cells, and the paths of integration within an elementary cell.

where $\Gamma_{\alpha\beta}$ is the total boundary of the elementary cell $V_{\alpha\beta}$. $\Gamma_{\alpha\beta}$ consists of the surfaces $\Gamma_{\alpha\beta}^{\text{cracks}}$ of microcracks penetrating the elementary cell, the boundaries $\Gamma_{\alpha\beta}^{\text{int}}$ between $V_{\alpha\beta}$, and the neighboring cells and a part $\Gamma_{\alpha\beta}^{\text{ext}}$ of external boundary ∂V when the elementary cell $V_{\alpha\beta}$ is one of the extreme peripheral cells of the active zone. When the summation in (14) is performed the integrals over $\Gamma_{\alpha\beta}^{\text{int}}$ cancel each other since there are always two opposite directions of integration. The sum of the integral over $\Gamma_{\alpha\beta}^{\text{ext}}$ results in the integral over the boundary of the active zone ∂V. The integrals over traction-free rectilinear microcrack surfaces are vanishing everywhere except the microcrack tips. There are two types of these integral paths, i.e. Γ^+ and Γ^- (see Fig. 4). The integral in the RHS of (15) over Γ^+ and Γ^- represent the energy release rates G_l^+ and G_l^- respectively. For small microcrack density one may employ a piecewise constant approximation of σ^{eff} on the scale of microcrack length l. It results in the following expression for ERR:

$$G_l^{\pm} = \pm\frac{\pi\langle l\rangle}{E}[(\mathbf{n}\sigma^{\text{eff}}\mathbf{n})^2+(\boldsymbol{\tau}\sigma^{\text{eff}}\mathbf{n})^2] \tag{16}$$

with + and − corresponding to the Γ^+ and Γ^-, respectively. In the total sum G_l^+ and G_l^- balance each other except when: (a) there is a balance in numbers of the "left" (−) and "right" (+) microcrack tips (see Fig. 4), and (b) there is a difference in the mathematical expectation of the microcrack length on the left and right size of the elementary cell under consideration. The first is associated with the gradient of the microcrack density ρ and the second with the gradient of the mathematical expectation of crack length $\langle l\rangle$ crossing a given point. The summation in (14) in the limit of N_α and N_β approaching infinity gives the final expression of ERR due to translation of the damage zone:

$$J_1 = \int_{\partial V} P_{1j}n_j\,d\Gamma - \int_V \frac{\pi}{E}\cos\theta\,[(\mathbf{n}\cdot\sigma^{\text{eff}}\cdot\mathbf{n})^2+(\boldsymbol{\tau}\cdot\sigma^{\text{eff}}\cdot\mathbf{n})^2][l(\cos\theta\,\partial_1\rho+\sin\theta\,\partial_2\rho) + \rho(\cos\theta\,\partial_1 l+\sin\theta\,\partial_2 l)]\,dv \tag{17a}$$

where θ is the average orientation of the microcracks.

Considering a high craze density case, we utilize the solution (Tada, 1973) for a craze in a thin strip of width $h = 1/\rho$ instead of piecewise constant approximation of σ^{eff}. The boundary condition is related to the craze formation stress σ^{cr} which is a material parameter. Then the ERR due to translation of the damage zone can be expressed as:

$$J_1 = \int_{\partial V} P_{1j}n_j\,d\Gamma - \int_V \frac{\cos\theta}{\rho E}\left[(\mathbf{n}\cdot\sigma^{\text{cr}}\mathbf{n})^2+\frac{E}{G}(\boldsymbol{\tau}\cdot\sigma^{\text{cr}}\mathbf{n})^2\right](\cos\theta\,\partial_1\rho+\sin\theta\,\partial_2\rho)\,dv. \tag{17b}$$

We have performed the computation of (17b) for the craze array reported by Botsis (1988). The total ERR J1 consists two parts, one path integral J_{1G} and one volume integral J_{1V}:

$$J_1 = J_{1\Gamma} + J_{1V} = 0.86G_1^0 + 0.54G_1^0 \tag{18}$$

where G_1^0 is the ERR of the main crack with no damage zone ($G_1^0 = (K_1^0)^2/E$). The volume integral depends on the craze formation stress σ^{cr} which is $1.6\sigma^{\infty}$ in above example.

4. SUMMARY

(1) The interaction between a main crack and a surrounding microcrack array is formulated in terms of the distributions of the microcrack density and the mathematical expectation of microcrack length. The formulation is based on the analytical solution of the interaction between a crack and a dislocation dipole. The approach is illustrated by a special case of a circular damage zone with a constant microcrack concentration and all microcracks being parallel to the main crack. The shielding effect of the microcrack array in this case is compared with that of an elastic inclusion. It is shown that an isotropic elastic inclusion model underestimates the shielding.

(2) In the second example a more realistic microcrack array configuration is considered. In this case the effective stress field within the damage zone is decomposed into singular and regular parts. This singular part with unknown effective stress intensity factor is taken from the asymptotic solution of a crack in an anisotropic material where anisotropy corresponds to effective elastic properties of the cracked material. The regular part of σ^{eff} is determined from the self-consistency equation.

(3) A new technique to evaluate the ERR associated with the damage zone translation is presented. The ERR consists of two parts. The first part is represented by a path integral similar to conventional J_1 integral. The second part is represented by an integral over the DZ domain and depends on the geometry of the process zone and the statistical distribution of microcrack density, length and orientation.

(4) The total ERR associated with translation of the damage zone is characterized by the microcrack density and length distributions. Computation of ERR for the particular craze array indicates that the ERR due to damage zone advance is the same order of magnitude as G_1^0.

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